Mismatch Errors in Microwave Phase Shift Measurements*

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Summary—**The phase difference between the incident and transmitted waves at the input and output ports, respectively, of a two-arm waveguide junction in a reflection free system is a characteristic of the waveguide junction and is defined as the "phase shift." The difference between the phase shift in a reflection free system and the "change of phase" observed in a system which is not reflection free will be termed mismatch error. The mismatch error depends not only on the reflections present in the system but also on the choice of the wave used as the reference wave in a phase measurement. Similar considerations hold for the measurements of variation of phase shift and the observed change of phase in adjustable components.**

A formal scattering matrix analysis is used to derive expressions for phase relationships of the wave amplitudes for a two-arm waveguide junction in a system with reflections. The results of this analysis are used to evaluate mismatch error for different choices of reference waves. Two techniques of variation of phase shift measurements are analyzed. Graphs of the limits of mismatch error in a commonly used method of measurement are presented.

Introduction

"THE phase shift through a waveguide component at a single frequency is the phase difference under matched conditions between corresponding incident and transmitted field quantities at the input and output ports, respectively, ignoring multiples of 2π radians."[1] From this definition, it is seen that the phase shift through a waveguide component is a characteristic of the component. However, if the component is inserted in a system which has reflections, two interactions take place which cause errors in measurements of phase shift. It will be shown that the phase difference between the emergent wave from the output port (transmitted wave) and the wave incident at the input port (incident wave) depends only on the reflection coefficient of the equivalent load attached to the junction and the characteristics of the junction. However, the phase of the incident wave with respect to some independent reference such as the component of the incident wave supplied by the generator depends on the reflection coefficients of both the load and the generator, and the characteristics of the junction. Consequently, the phase of the emergent wave with respect to an independent reference depends on the reflection coefficients of the load and generator and characteristics of the junction. The difference between the phase shift and the phase change observed will be termed a mismatch error. Care must be exercised to determine which wave is being used as a reference in evaluating these mismatch errors. Similar considerations hold for measurements of variation of phase shift and the observed change of phase in adjustable components such as microwave phase shifters or attenuators.

A scattering matrix analysis is used to derive the phase relationship among various wave amplitudes in a two-arm waveguide junction inserted in a system with reflections. Mismatch errors are evaluated for two choices of reference waves. Two commonly used methods of measuring variations of phase shift in adjustable components are analyzed for mismatch error. Limits of mismatch error are calculated for the first method, a two channel arrangement, and presented in two graphs. One graph presents limits of error for lossless components and is valid for low loss phase shifters. The limits of mismatch error for a lossless phase shifter are slightly larger than those which would be encountered in a low loss component such as a commercial phase shifter, or in an attenuator when one or both of the settings is less than 20 db. The other graph is for components which have at least 20 db loss at both settings. This graph is presented since such measurements have smaller limits of mismatch error.

The second method which is treated uses a short circuit and slotted line to measure the phase shift or variation of phase shift of low-loss components. The error is evaluated and it is found to depend to the first order only on the mismatches of the component and not on the mismatches of the generator.

Theory

A two-arm waveguide junction may be represented as in Fig. 1. The phase of the emergent wave from arm 2

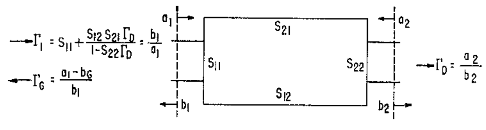

Fig. 1—A two-arm waveguide junction representation.

(the transmitted wave) with respect to the other waves associated with the junction may be derived by the use of the scattering matrix, S. In terms of this matrix,

$$b = Sa \tag{1}$$

where b is a column matrix of the emergent wave ampli-

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[1] "IRE standards on antennas and waveguides: waveguide and waveguide component measurements, 1959," PROC. IRE, vol. 47, pp. 568–582; April, 1959.

tudes, a is a column matrix of the incident wave amplitudes, and S is the scattering matrix of the junction. It can readily be shown from (1) and Fig. 1 that the output wave is related to the component of the input wave supplied by the generator b_G by

$$\frac{b_2}{b_G} = \frac{S_{21}}{(1 - \Gamma_1\Gamma_G)(1 - S_{22}\Gamma_D)} \tag{2}$$

where the S's are elements of the scattering matrix associated with the two arm junction, and Γ_G and Γ_D are the equivalent generator and detector reflection coefficients, respectively, and Γ_1 is the reflection coefficient of the equivalent load attached to the generator. Γ_1 may be expressed as

$$\Gamma_1 = S_{11} + \frac{S_{12}S_{21}\Gamma_D}{1 - S_{22}\Gamma_D}. \tag{3}$$

The argument of (2) is the phase difference between the emergent wave, b_2, and b_G, which is the wave that would be delivered to a reflectionless load. b_G is independent of the reflections of the system and therefore is termed the independent wave.

Eq. (2) may be written in the form

$$\frac{b_2}{b_G} = \frac{b_2}{a_1}\cdot\frac{a_1}{b_G} \tag{4}$$

where

$$\frac{b_2}{a_1} = \frac{S_{21}}{1 - S_{22}\Gamma_D} \tag{5}$$

and

$$\frac{a_1}{b_G} = \frac{1}{(1 - \Gamma_1\Gamma_G)}. \tag{6}$$

The argument of b_2/a_1 is the phase difference between the transmitted wave and the incident wave. The argument of a_1/b_G is the phase difference between the incident wave and the independent generator wave, b_G. When $\Gamma_D = \Gamma_G = 0$, the phase difference of (6) reduces to zero and both (2) and (5) reduce to

$$\frac{b_2}{a_1} = S_{21} = |S_{21}|\, e^{j\phi_{21}} \tag{7}$$

where ϕ_{21} is, by definition, the phase shift through the waveguide component.

Evaluation of Mismatch Error

Case I. The Reference Wave is the Independent Wave, b_G

In techniques where the independent wave is used as the reference wave, the mismatch error for a phase shift measurement may be obtained by rewriting (2) in the form

$$\frac{b_2}{b_G} = |S_{21}|\, e^{j\phi_{21}}\, |E_a|\, e^{j\epsilon_a}, \tag{8}$$

where

$$|E_a|\, e^{j\epsilon_a} = \frac{1}{(1 - \Gamma_1\Gamma_G)(1 - S_{22}\Gamma_D)}. \tag{9}$$

The difference between the measured change of phase and the phase shift of the component is just ϵ_a, the argument of (9). For differential phase shifters or attenuators, using front superscripts i and f to denote initial and final settings, respectively, the change of phase of b_2 with respect to the independent wave may be obtained from an expression derived from (2), which is

$$\frac{{}^f b_2}{{}^i b_2} = \frac{{}^f S_{21}}{{}^i S_{21}}\, \frac{(1 - {}^i\Gamma_1\Gamma_G)(1 - {}^i S_{22}\Gamma_D)}{(1 - {}^f\Gamma_1\Gamma_G)(1 - {}^f S_{22}\Gamma_D)}, \tag{10}$$

where the argument of (10) is the change in phase of the emergent wave, b_2, with respect to the independent wave when the setting of the junction is changed. For $\Gamma_D = \Gamma_G = 0$, this reduces to

$$\frac{{}^f b_2}{{}^i b_2} = \frac{{}^f S_{21}}{{}^i S_{22}} = \frac{|{}^f S_{21}|}{|{}^i S_{21}|}\, e^{j({}^f\phi_{21} - {}^i\phi_{21})}, \tag{11}$$

where ${}^f\phi_{21} - {}^i\phi_{21}$ is the variation of phase shift when the setting of the junction is changed. For Γ_G and Γ_D not zero, (10) may be written in the form

$$\frac{|{}^f b_2|}{|{}^i b_2|}\, e^{j({}^f\psi_2 - {}^i\psi_2)} = \frac{|{}^f S_{21}|}{|{}^i S_{21}|}\, e^{j({}^f\phi_{21} - {}^i\phi_{21})}\, |E_b|\, e^{j\epsilon_b}, \tag{12}$$

where ${}^f\psi_2 - {}^i\psi_2$ is the change in phase of the emergent wave for Γ_D and Γ_G not zero, ${}^f\phi_{21} - {}^i\phi_{21}$ is the variation of phase shift of the component, and

$$E_b = |E_b|\, e^{j\epsilon_b} = \frac{(1 - {}^i\Gamma_1\Gamma_G)(1 - {}^i S_{22}\Gamma_D)}{(1 - {}^f\Gamma_1\Gamma_G)(1 - {}^f S_{22}\Gamma_D)}. \tag{13}$$

From (12) it can be seen that the mismatch error in this case is the argument of (13).

Case II. The Reference Wave is the Incident Wave at the Input, a_1

In techniques where the incident wave is used as the reference wave, (5) may be written in the form

$$\frac{b_2}{a_1} = |S_{21}|\, e^{j\phi_{21}}\, |E_c|\, e^{j\epsilon_c} \tag{14}$$

where ${}^f\phi_{21}$ is the phase shift of the component and

$$|E_c|\, e^{j\epsilon_c} = \frac{1}{1 - S_{22}\Gamma_D}. \tag{15}$$

It can be seen that the argument of (15) is the mismatch error when the incident wave is used as a reference. For adjustable components, the change in phase of the emergent wave may be written, when the incident input wave is used as a reference wave, as

$$\frac{{}^f b_2}{{}^i b_2} = \frac{{}^f S_{21}}{{}^i S_{21}}\, \frac{1 - {}^i S_{22}\Gamma_D}{1 - {}^f S_{22}\Gamma_D} \tag{16}$$

or as

$$\frac{{}^f b_2}{{}^i b_2} = \frac{|{}^f S_{21}|}{|{}^i S_{21}|} e^{j({}^f\phi_{21} - {}^i\phi_{21})} \, |E_d| \, e^{j\epsilon_d} \tag{17}$$

where

$$|E_d| \, e^{j\epsilon_d} = \frac{1 - {}^i S_{22}\Gamma_D}{1 - {}^f S_{22}\Gamma_D} \tag{18}$$

and ϵ_d is the mismatch error in this case.

Application I

A two-channel method of measuring variation of phase shift is illustrated in Fig. 2. Usually there is considerable isolation between the component under test and power dividing network. Under these circumstances, a portion of the energy of the oscillator traverses a separate isolated path and behaves as an independent reference wave for the change of phase measurements.

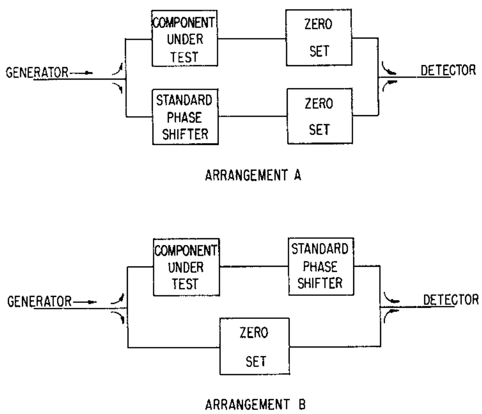

Fig. 2—Arrangement of equipment for a two-path method of measuring phase shift.

Without this isolation, the equipment may be adjusted to use the incident wave as a reference. The present discussion assumes this isolation to be infinite. Since the independent wave is used as a reference, the mismatch error is given by (13). If the magnitudes of the terms other than $|E|$ and unity in (13) are small compared to unity then to a good approximation,

$$|E| \, e^{j\epsilon} = 1 - {}^i\Gamma_1\Gamma_G - {}^i S_{22}\Gamma_D + {}^f\Gamma_1\Gamma_G + {}^f S_{22}\Gamma_D, \tag{19}$$

and the mismatch error, ϵ, may be written approximately as

$$\epsilon = \text{argument of } [1 - {}^i\Gamma_1\Gamma_G - {}^i S_{22}\Gamma_D + {}^f\Gamma_1\Gamma_G + {}^f S_{22}\Gamma_D]. \tag{20}$$

However, it is inconvenient or sometimes difficult to evaluate the phases of the scattering and reflection coefficients, while limits of their magnitudes are more readily determined from estimates of maximum VSWR. Therefore, limits of error (maximum error) for arbitrary phases of these coefficients are evaluated here. One may represent (19) in graphical form as shown in Fig. 3. Allowing the phases of the coefficients to take on appropriate values, the maximum error, lim ϵ, will occur when the resultant is 90° out of phase with the variables, as shown in Fig. 4. Under these conditions

$$\sin(\lim \epsilon) = |{}^i\Gamma_1\Gamma_G| + |{}^i S_{22}\Gamma_D| + |{}^f\Gamma_1\Gamma_D| + |{}^f S_{22}\Gamma_D|, \tag{21}$$

which for small angles may be written,

$$\lim \epsilon = |{}^i\Gamma_1\Gamma_G| + |{}^i S_{22}\Gamma_G| + |{}^f\Gamma_1\Gamma_G| + |{}^f S_{22}\Gamma_D|. \tag{22}$$

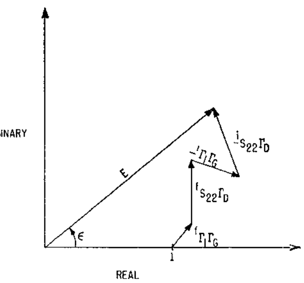

Fig. 3—Representation of (19).

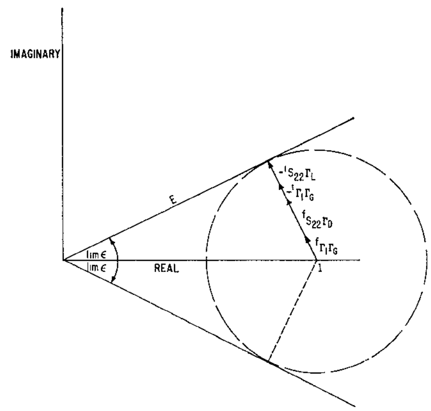

Fig. 4—Representation of (19) for maximum ϵ.

A conservative estimate of the limits of mismatch error may be quickly obtained by using the manufacturers' specifications for the magnitudes of the scattering coefficients. It should be noted, however, that for a specific measurement, determining the value of the magnitude of the scattering coefficients will usually result in smaller limits of error since manufacturers generally specify only the maximum value over the entire operating range. It should be emphasized that the limits of error calculated from (22) are maximum errors based on the assumption that the phases of the scattering coefficients change an arbitrary amount. Limits to these phase changes can frequently be determined, and for precise measurements, it is then desirable to use these limits and determine the smaller limits of error by use of (13).

Graphical Presentation of Results

The graphs are constructed to present a limit of mismatch error in variation of phase shift measurements as a function of the mismatches of the generator, detector, and phase shifter. The following assumptions were made to simplify the presentation and they introduce only a small loss of generality. It is assumed that: 1) the equivalent generator and detector reflection coefficients are of equal magnitude, $|\Gamma_G| = |\Gamma_D|$; 2) the input and output voltage standing-wave ratio (VSWR) of the phase shifter are equal, and therefore $|S_{11}| = |S_{22}|$; and 3) the detector reflection coefficient and S_{11} combine to give maximum magnitude of Γ_1.

The results for a lossless phase shifter are presented in Fig. 5, and for a lossy one in Fig. 6. The limits of error are plotted against the input or output VSWR

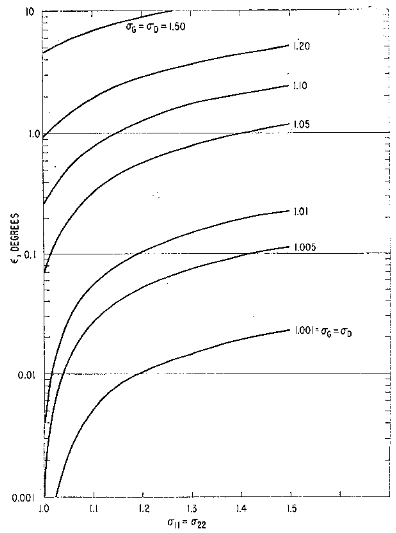

Fig. 5—Limit of mismatch error for lossless phase shifters.

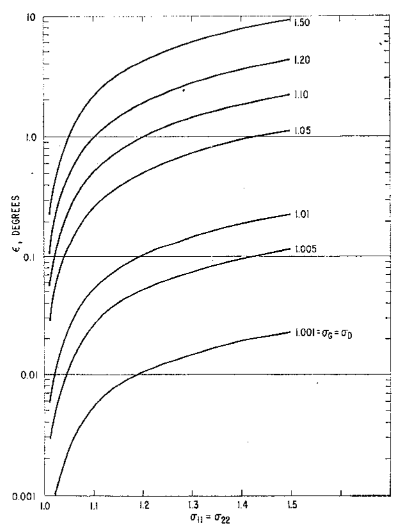

Fig. 6—Limit of mismatch error for attenuators with initial and final settings greater than 20 db.

(σ_{11} or σ_{22}) of the phase shifter or attenuator. Fig. 5 gives limits of error for a lossless phase shifter and conservative limits of error for phase shifters with less than 20 db loss or attenuators with one or both settings at less than 20 db loss. Fig. 6 gives limits of error for attenuators when both settings are at least 20 db or phase shifters with 20 db or more of insertion loss.

As an example, consider a phase shifter with 0.5 db insertion loss and maximum input and output VSWR of 1.35 placed in a waveguide system with maximum VSWR looking towards the generator and detector of 1.05. The conservative limit of error as given by Fig. 5 is 0.90°. However, if this component has 20 db or more loss at both settings, the limit of error as given by Fig. 6 is 0.84°. The difference between the limits of error for the lossless and high loss cases becomes more pronounced as the ratio of $|\Gamma_D|$ to $|S_{11}|$ becomes larger and therefore both graphs are presented.

The graphs may also be used to estimate the maximum permissible VSWR of the equivalent generator and detector to attain a given accuracy of variation of phase shift with a calibrated phase shifter. One case of interest is a microwave phase shifter of maximum VSWR of 1.35 which is calibrated to 2° accuracy. To utilize this accuracy, an estimate from Fig. 5 indicates that it should be used in a system where the VSWR looking towards the generator and detector are 1.10, or less. Another case of interest is the comparison of the variation of phase shift of two components within 0.1° in a two channel method. This would be satisfied if the limit of mismatch error for each component was 0.05°. If one of the components is a microwave phase shifter with maximum VSWR of 1.35, the maximum VSWR of the equivalent generator and detector for 0.05° limit is 1.004. If the other component is an attenuator with maximum VSWR of 1.15, the maximum VSWR of the equivalent generator and detector for 0.05° limit is 1.006.

It may be useful here to emphasize the meaning of the limits of error presented in the graphs. These are maximum errors due to mismatches, since it was assumed that the phase changes of all coefficients were arbitrary. If one has knowledge of the limits of phase changes of the coefficients, or actual values, it is desirable for critical work to turn to (11) and evaluate closer limits of mismatch error, or actual mismatch error.

Application II

A method which has been used to measure the variation of phase shift of a low-loss reciprocal waveguide component by terminating it with a calibrated sliding short circuit and using a slotted section as a detector is illustrated in Fig. 7. A minimum of the input standing wave pattern is used as a reference to position the probe. When the component under test is adjusted to a new setting, the minimum of the input standing wave pattern is restored to the reference plane of the probe by moving the calibrated short circuit. Since the wave travels through the component in both directions to form this pattern, twice the phase shift of the component is assumed to be equal to the change of phase of the reflection coefficient of the attached short circuit.

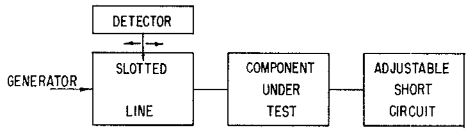

Fig. 7—Arrangement of equipment for a short circuit method of measuring phase shift of low-loss components.

Neglecting any errors caused by probe loading, the minimum of the input standing wave pattern occurs when

$$\text{argument of } \frac{b_1}{a_1} = (2n+1)\pi, \tag{23}$$

where n is an integer. The minimum of the pattern is restored to the initial position by adjustment of the short circuit and this condition may be expressed by

$$\arg \frac{{}^fb_1}{{}^fa_1} = \arg \frac{{}^ib_1}{{}^ia_1}, \tag{24}$$

or

$$\arg {}^f\Gamma_1 = \arg {}^i\Gamma_1 \tag{25}$$

where Γ_1 is the input reflection coefficient of the component when arm 2 is terminated with a sliding short circuit with reflection coefficient Γ_s. Substitution of an expression for Γ_1 in terms of the scattering coefficients of the component and the short circuit allows the adjustment conditions to be written as

$$\arg\left({}^fS_{11} + \frac{{}^fS_{21}{}^2\,{}^f\Gamma_s}{1 - {}^fS_{22}{}^f\Gamma_s}\right) = \arg\left({}^iS_{11} + \frac{{}^iS_{21}{}^2\,{}^i\Gamma_s}{1 - {}^iS_{22}\,{}^f\Gamma_s}\right), \tag{26}$$

when reciprocity in the form, $S_{21}=S_{12}$, has been assumed. For $|S_{22}\Gamma_s| \ll 1$, this may be written approximately as

$$\arg ({}^fS_{11} + {}^fS_{21}{}^2\,{}^f\Gamma_s + {}^fS_{21}{}^2\,{}^fS_{22}\,{}^f\Gamma_s{}^2) = \arg ({}^iS_{11} + {}^iS_{21}{}^2\,{}^i\Gamma_s + {}^iS_{21}{}^2\,{}^iS_{22}\,{}^i\Gamma_s{}^2). \tag{27}$$

The measured variation of phase shift is based on the assumption that $S_{11}=S_{22}=0$ and that the arguments of ${}^fS_{21}{}^2\,{}^f\Gamma_s$ and ${}^iS_{21}{}^2\,{}^i\Gamma_s$ are equal, which leads to

$${}^f\phi_{21} - {}^i\phi_{21} = \tfrac{1}{2}({}^i\psi_s - {}^f\psi_s) \tag{28}$$

where ${}^i\psi_s$ and ${}^f\psi_s$ are the initial and final phases, respectively, of the reflection coefficient of the sliding short circuit, Γ_s.

It is apparent, however, that the actual change of phase can differ from this ideal when S_{11} and S_{22} are not zero, or

$$({}^f\phi_{21} - {}^i\phi_{21}) - \tfrac{1}{2}({}^i\psi_s - {}^f\psi_s) = \epsilon_e, \tag{29}$$

where ϵ_e is the mismatch error in this method. Eq. (29) can be shown to be equivalent to

$$2\epsilon_e = \arg \frac{{}^fS_{21}{}^2\,{}^f\Gamma_s}{{}^iS_{21}{}^2\,{}^i\Gamma_s} = \arg {}^fS_{21}{}^2\,{}^f\Gamma_s - \arg {}^iS_{21}{}^2\,{}^i\Gamma_s. \tag{30}$$

The difference between these two arguments, $2\epsilon_e$, can be seen from Fig. 8, a graphical representation of (27) which describes the actual adjustment of conditions for the general case. The limits of this difference, assuming all phases of the reflection coefficients are possible, may be obtained from

$$\begin{aligned}\sin (\lim \epsilon_e) &= \frac{1}{2}\,[\sin(\lim\epsilon_1) + \sin(\lim\epsilon_2)] \\ &= \frac{1}{2}\,\frac{|{}^fS_{11}| + |{}^fS_{21}{}^2\,{}^fS_{22}\,{}^f\Gamma_s{}^2|}{|{}^fS_{21}{}^2\,{}^f\Gamma_s|} \\ &\quad + \frac{1}{2}\,\frac{|{}^iS_{11}| + |{}^iS_{21}{}^2\,{}^iS_{22}\,{}^i\Gamma_s{}^2|}{|{}^iS_{21}{}^2\,{}^i\Gamma_s|}\end{aligned} \tag{31}$$

where $\lim \epsilon_e$ is the limit of error, and $\lim \epsilon_1$ and $\lim \epsilon_2$ are limits of ϵ_1 and ϵ_2 as shown in Fig. 8. A readily calculated approximation for the limit of error may be found by assuming $|S_{21}|=1$, $|\Gamma_s|=1$, and $|S_{11}|$ and $|S_{22}|$ do not change with adjustment. This approximate limit of error may be obtained from

$$\sin (\lim \epsilon_e) \approx |S_{22}| + |S_{11}|. \tag{32}$$

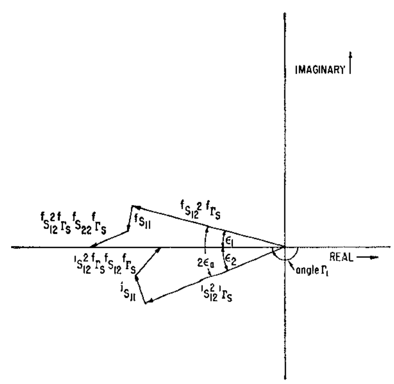

Fig. 8—Representation of (27).

If the input and output VSWR of the component are equal and the error is small, the limit of mismatch error may be written as

$$\lim \epsilon_e = 2 | S_{11} | \text{ radians} \tag{33}$$

which can be readily shown to be equivalent to a result for lossless components quoted by Magid.[2] It may be useful here to emphasize the meaning of the limits of error obtained by (33). These are maximum errors due to mismatches, since it was assumed that the phase changes of all coefficients were arbitrary.

[2] M. Magid, "Precision microwave phase shift measurements," IRE Trans. on Instrumentation, vol. I-7, pp. 321–331; December, 1958.

For the same phase shifter considered in Application I (VSWR < 1.35), the limits of error are ±17°. It is of interest to note that the mismatch error in a variation of phase shift measurement in this method is independent of the reflection coefficient of the generator.

Additional errors in this method such as those caused by probe loading in the slotted line are not within the scope of this analysis, but should be taken into account, if they are appreciable compared to the mismatch error.

Acknowledgment

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A Note on the Optimum Source Conductance of Crystal Mixers*

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Summary—**This paper describes an accurate and convenient technique for measuring the match of a crystal mixer. Use is made of the fact that with a proper RF drive level, the fundamental conductance of a mixer crystal may be made equal to the optimum source conductance of the crystal for mixer operation. The required drive level depends on certain crystal parameters and on the image frequency termination of the mixer. Design curves are given which simplify the determination of the proper RF drive level for a wide range of crystal parameters and their condition of image frequency termination.**

Introduction

The design of a crystal mixer may conveniently be broken down into three parts:

1) design of a signal coupling mechanism which will provide the optimum source conductance for minimum available conversion loss,
2) design of a local-oscillator coupling mechanism that has negligible effect on the signal admittance,
3) design of an RF bypass circuit that will not allow the RF power to couple to the IF load circuit.

This paper deals with certain considerations of the design of the signal coupling mechanism. In order to achieve a minimum noise figure crystal mixer receiver, it is necessary to design the mixer for minimum available conversion loss. From linear network theory, applicable to a crystal mixer, there is an optimum source conductance for minimum available conversion loss. In the practical design of a crystal mixer, it is usually assumed that this optimum conductance is equal to the fundamental component of the conductance of the crystal for a high-level RF signal, of the same magnitude as the local-oscillator drive, but in the absence of this local-oscillator drive. Under this assumption, the crystal mount is then designed to be matched to the line at this high level of RF signal.[1] This generally gives a good approximation to the optimum match condition for the broadband mixer[2,3] (image frequency termination equal

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[1] R. V. Pound, "Microwave Mixers," Rad. Lab. Series, McGraw-Hill Book Co., Inc., New York, N. Y., vol. 16, p. 122; 1945.
[2] H. C. Torrey and C. A. Whitmer, "Crystal Rectifiers," Rad. Lab Series, McGraw-Hill Book Co., Inc., New York, N. Y., vol. 15, pp. 111–178; 1948.
[3] Operation in this condition is discussed by Torrey and Whitmer, *Ibid.* Data cited there show that the conversion loss (L_0) for this condition differs from the optimum conversion loss (L_2) in the broadband condition by less than 0.2 db.